The Trick that Makes Physics as Simple as Drawing a Picture

Problem Sheet

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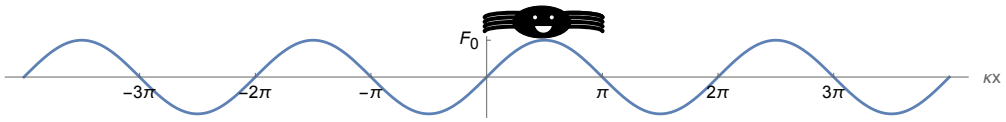
Sally the spider is walking along her merry way when she is suddenly caught in a mysterious force field! The force is given by $F(x) = F_0 \sin(\kappa x)$, where F_0 and κ are constants, plotted below:



(**a**) Identify all the equilibrium points in the plotted potential. Which are stable and which are unstable?

(**b**) Find and draw the potential energy $U(x)$. Hint: the slope of $\sin(\kappa x)$ is $\kappa\cos(\kappa x)$, and the slope of $\cos(\kappa x)$ is $-\kappa\sin(\kappa x)$.

Suppose Sally is at position $x_0 = \frac{\pi}{\kappa}$ and has velocity v_0 at $t = 0$. She bundles herself up into a little ball like spiders do when they're scared, and coasts along without friction.

(**c**) Write the $F = ma$ equation for Sally's position $x(t)$. Under what circumstances do you expect it to have a simple solution?

(**d**) Describe the behavior of Sally's trajectory $x(t)$ by thinking about the shape of the potential. Be sure to distinguish the different cases, depending on how big v_0 is.

(**e**) What will be the maximum x coordinate that Sally reaches for $t > 0$?

(**f**) Suppose v_0 is small enough that Sally doesn't get very far away from $\kappa x = \pi$. Write the explicit solution of the $F = ma$ equation in that case, using the fact that $\sin(\kappa x) \approx -(\kappa x - \pi)$. Is your solution consistent with what you expected from the shape of the potential?