Everything You Need To Know About Pendulums

Problem Sheet

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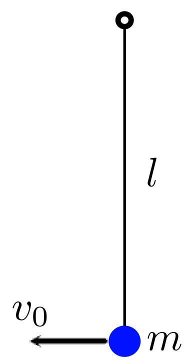
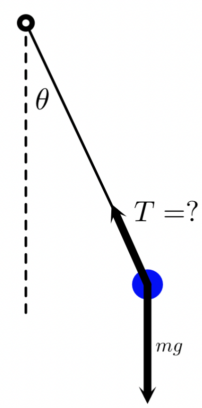
Solutions available at PhysicsWithElliot.com/pendulum-help-room

1



You give a pendulum, which was initially at rest in equilibrium, a tap to the left at $t = 0$ that gives it initial speed v_0. Assuming that you tapped it gently enough that the pendulum doesn't swing too far away from equilibrium, determine $\theta(t)$ for $t > 0$. How big can v_0 be before you can't trust your solution anymore?

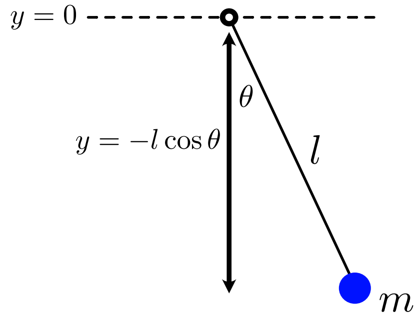
2



Determine the tension T in the rod of a simple pendulum as a function of θ and $\dot{\theta}$. Check that your answer is consistent with what you expect when the pendulum is at rest in equilibrium. Evaluate the tension as a function of time for a pendulum that's released from rest from a small initial angle θ_0 at $t = 0$.

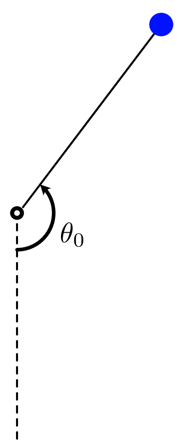
Hint: Since the particle is moving around in a circle, the total force pointing toward the center has to supply the **centripetal** force $m\dot{\theta}^2 l$ that's always required to keep a mass in circular motion.

3



The kinetic energy of an oscillating pendulum is $K(t) = \frac{1}{2}m\dot{s}^2 = \frac{1}{2}ml^2\dot{\theta}^2$, and the potential energy is $U(t) = mgy$, where $y = -l\cos\theta$ is the height of the particle (where I've called the height of the pivot $y = 0$). Show that the total energy $E = K(t) + U(t)$ is a constant, independent of time.

4



A pendulum is released from rest from an initial angle θ_0, which isn't necessarily a small number. Determine the maximum speed that the particle reaches. Check that your result is consistent with what you expect when θ_0 *is* a small number. Hints: Use energy conservation! You'll also need the small angle approximation for cosine, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$ when θ is small.

5 For a pendulum that's released from rest from a small angle θ_0, we saw that the period is $T = 2\pi/\Omega$, where $\Omega = \sqrt{g/l}$ is the natural frequency. Show that the exact period—not assuming θ_0 is small—may be expressed as

$$T = \frac{2\sqrt{2}}{\Omega}\int_0^{\theta_0} \frac{\mathrm{d}\theta}{\sqrt{\cos\theta - \cos\theta_0}},$$

and show that this reduces to $2\pi/\Omega$ when θ_0 is small. The period for small angles was independent of θ_0; is the same true for large θ_0's? Hints: Use energy conservation to solve for the angular velocity $\dot{\theta}$. Then use the identity $\frac{\mathrm{d}\theta}{\dot{\theta}} = \mathrm{d}t$ and integrate both sides over one full oscillation. The right-hand-side is the period, and the left-hand-side is what you're looking for.